A Theory of the Supersonic Turbulent Axisymmetric Near Wake behind Bluff-Base Bodies

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Theme

ONE of the current interests in solving the near-wake problem in a supersonic flow is to predict the effect of various base drag reducing concepts. The external burning concept, as applied by Strahle[1] to the two-dimensional case using the simple Crocco-Lees[2] theory showed that significant base drag reduction can be obtained using this concept. The motivation of the present investigation was to apply this concept to a more practical projectile shape, viz., axisymmetric, in a more quantitatively exact way. This now is warranted because of the notable improvement in the Crocco-Lees approach by Reeves and Lees[3] for the laminar case and the remarkably successful application of the Reeves and Lees analysis by Alber and Lees[4] to the two-dimensional turbulent case, using an eddy viscosity concept. Here, Alber and Lees' work, with suitable modifications, has been extended to the axisymmetric case. The present theory is for the adiabatic axisymmetric supersonic turbulent near wake without combustion, but contains sufficient flexibility to incorporate simple external combustion. The reversed flow has been treated with sufficient detail so as to allow for base bleed, because reactive bleed may be a promising adjunct to external burning to improve performance. Also, an entropy layer in the inviscid stream is allowed for, because several concepts of external burning would introduce the fuel by injection into the supersonic stream, causing injection shocks.

Contents

Complete details of the present work and the references are given in Ref. 5. The flow model used for the present analysis is as shown in Fig. 1. The lip shock and the lip-shock/wake-shock interaction have been neglected, since they have been shown by previous studies to be of secondary importance at moderate Mach numbers.

The analysis essentially consists of solving the corner flow region and the flow downstream of the base. The solution of the corner region provides the initial conditions for the wake analysis. Earlier work showed that after such extreme expansions as occurring in the present case, viscous forces remain predominant only in a small part Δ_2 of the expanded boundary layer, corresponding to the portion Δ_l of the initial boundary layer δ_l. These thicknesses are found by assuming a 1/7th power profile for the initial boundary layer, a value for the base pressure, $(\partial u/\partial r)_{\Delta_2} = (\partial u/\partial r)_{\delta_l}$, and an isentropic expansion in the outer streamtubes, for given upstream Mach number M_l, pressure p_l, momentum thickness θ_l, and base radius R.

The flow downstream of the base, as shown in Fig. 1, consists of four regions. The inner regime, consisting of the recirculatory region [1] and shear flow region [2], is represented by integrated boundary-layer equations. The turbulent shear stress is evaluated by following an approach similar to that of Alber and Lees. An eddy viscosity model is chosen differently for the region upstream of the RSP and downstream of the RSP. The constant, and the characteristic length and velocity scales in the incompressible eddy viscosity models are defined to recover the well-known results of the self-preserving shear layer and the far wake, and to give a continuous variation of the viscosity at the RSP. This incompressible eddy viscosity then is multiplied by a compressibility factor to obtain the proper Mach number dependence.

The solution from the base is started using the Green[6] two-parameter incompressible velocity profiles, which contain a constant velocity core. The use of these profiles results in six unknowns in the problem, for an assumed value of the base pressure. The mass conservation equation, the momentum conservation equation, the mechanical energy equation, and the centerline momentum equation, in conjunction with the solution of the outer regime, are used to solve for these unknowns. The outer regime consists of the inviscid rotational region [3] and the inviscid isentropic region [4]. Consistent with the accuracy and the spirit of the present approach, this regime is solved by an approximate method of characteristics (AMC) by modifying Webb's[7] method.

As one moves downstream of the base, the initial constant velocity core is eaten up by the shear layer, and the number of parameters is reduced by one. Better matching of the maximum reverse velocity with the experiments takes place if a small constant velocity core in the velocity profiles is carried through downstream, or the Kubota et al.[8] one-parameter profiles are allowed to take over. However, it is not always possible to match the starting Green's profiles with the Kubota et al. profiles. Anyway, the unknowns are reduced by one at a unique matching point. The solution of the three integrated equations and the outer regime is sufficient to calculate the five unknowns. The solution is continued downstream until the Crocco-Lees singularity is hit. If the initial value of base pressure guessed is low, the determinant of the solution matrix goes through zero before any of the numerators and vice versa. It requires from 3 to 10 sec to complete one iteration on a CDC CYBER 70-74.

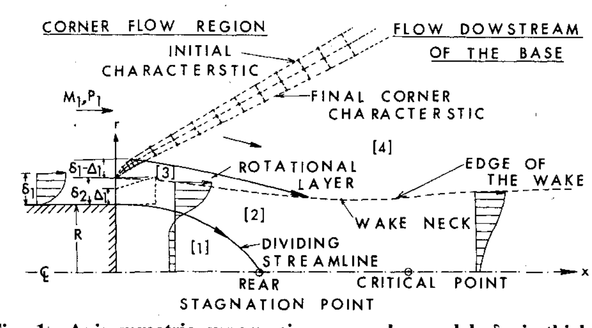

Fig. 1 Axisymmetric supersonic near wake model; δ_2 is thickness corresponding to Δ_2 after separation.

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Index categories: Jets, Wakes, and Viscid-Inviscid Flow Interactions; Supersonic and Hypersonic Flow; Viscous Nonboundary-Layer Flows.

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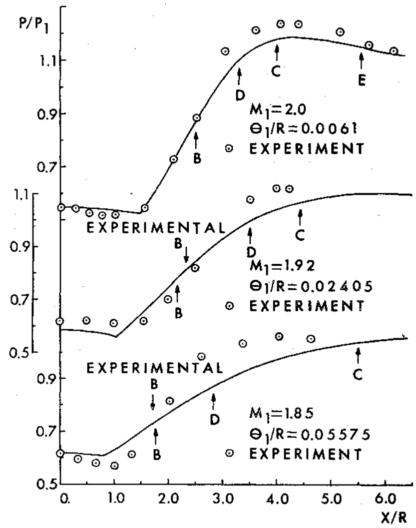

Fig. 2 Centerline pressure variation with axial distance and comparison with experiments near Mach 2; B = rear stagnation point, C = Crocco-Lees critical point, D = wake neck, E = centerline Mach 1 point.

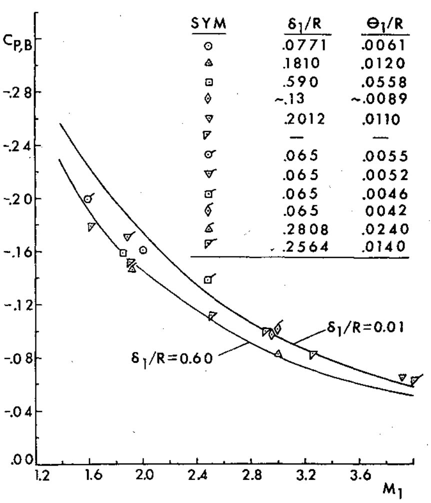

Fig. 3 Effects of Mach number on base pressure coefficient.

Figures 2 and 3 show a comparison of some of the results of the present theory with the experimental data, and, as seen, good agreement is obtained. However, theory seems to slightly overpredict the base pressure at large Mach numbers (~above 2.5) and slightly underpredict the base pressure at low Mach numbers (~below 1.8). It is found that, if the eddy viscosity is multiplied by a factor $\sqrt{M_l/2}$, a much better correlation with the experimental results can be obtained for a wider range of Mach number. Also, the variation of centerline Mach number and the effect of boundary-layer thickness on base pressure compared well with experiments. These comparisons with the experiments thus show the adequacy of the present modeling of the corner region and the shear stress. A number of studies, using the present program, were made and the major conclusions arrived at are as follows:

1) The solution of outer regime plays a very important role in the present problem; the replacement of AMC with the Prandtl-Meyer relation resulted in very poor results.

2) For better prediction of flowfield details, a good velocity profile is essential. However, base pressure is affected in a minor way because of a change in profiles.

3) The base pressure is a strong function of upstream Mach number and decreases with an increase of Mach number. The detailed quantities, such as location of the RSP, velocity on dividing streamline, etc., are only weak functions of Mach number.

4) The base pressure is a weak function of the upstream boundary-layer thickness. All other quantities such as centerline Mach number, etc. are strong functions of δ_l. Hence, measurement of the latter quantities can shed more light on and help in improving the corner model.

5) At low base bleed rates, the theory shows a much smaller base pressure rise compared with experiments. The inaccuracy of the boundary-layer equation, in representing the region close to the base, may be the major reason for this result.

In conclusion, the method developed here is computationally fast, provides adequate details of the near wake, and hence should be quite useful for preliminary design purposes.

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